HIDDEN MOVES AND RISKY CHOICES

Concepts and techniques

- Hidden moves
- Risk
- Expected value
- Expected utility
- Attitudes to risk
- Independence axiom
- Common consequence and common ratio effects
- Transitivity
- Framing, preference reversal.

After working through this chapter you will be able to:

- Analyse a game with hidden moves
- Illustrate hidden moves in the extensive form of a dynamic game
- Use expected values and expected utility to calculate expected pay-offs
- Use examples to show how attitudes to risk affect choices between risky options
- Outline the theoretical limitations of expected utility theory
- Explain what is implied by the independence axiom of expected utility theory
In Section 1.8 of Chapter 1 it was stated that the outcome of a game will depend on the information that the players have. In the games considered so far, all the players have had the same information, they knew where they were in the game and who they were playing. In this chapter you will see how to model situations where there is less shared information than this. First, you will see how simultaneous-move games can be modelled as dynamic games with hidden moves. I have already claimed that these two possibilities are equivalent in game theoretic terms and in Section 5.1 you will see why this is the case. In Section 5.2, risk is modelled by incorporating probabilities in individual decision-making problems. Using probabilities allows the analyst to calculate either expected values or expected utilities. The latter usage is generally considered the more versatile but expected utility theory is not without its critics. Some of the criticisms of expected utility theory are supported by experimental evidence and this will be examined in detail in Section 5.3.

We will start by considering the battle of the sexes game in Matrix 5.1. This game was first analysed in Chapter 2 in Matrix 2.19 and you should verify that it has two Nash equilibria in pure strategies: \{pub, pub\} and \{party, party\}. In Chapter 2 the players were assumed to move simultaneously or if the players moved at different times their moves were hidden. I've claimed that these two possibilities are equivalent and intuitively it is not difficult to see why this might be; if a move is hidden to a player then it cannot really matter when it was made. Here we can use the methodology of Chapter 4 to argue this point more forcibly. We will do this by analysing a sequential-move version of the battle of the sexes game. Two possibilities are considered: a sequential-move game with seen moves and one with hidden moves.
Let's first consider the battle of the sexes game as a sequential-move game in which John moves first but there are no hidden moves. Figure 5.1 illustrates this case; John moves first and Janet observes John's move. In this version of the game if John chooses pub Janet makes a decision at Janet₁ and if he chooses party then she makes a decision at Janet₂. Because Janet's move is contingent on John's she has four pure strategies: (pub, pub), (pub, party), (party, pub) and (party, party). The players' pay-offs are such that if John chooses pub, so will Janet, but if John chooses party she will choose party. Thus only (pub, party) is rational for Janet. John knows this and as he prefers the pub the subgame perfect Nash equilibrium of this dynamic version of battle of the sexes is \{pub, (pub, party)\} which implies that both of them go to the pub. This is what I meant in Chapter 2 Section 2.4.3 when I said that the game had a first mover advantage; if the moves are seen, then whoever moves first can secure their preferred outcome. To show that this is true when Janet has the first move draw the game tree with Janet moving first. You should be able to use backward induction to argue that the subgame perfect Nash equilibrium now has both players going to the party. This shows that, unlike the simultaneous-move game, there is a unique equilibrium outcome when one of the players moves first and their move is observed by the other. This is not the case when the player's move is hidden.

We can see this by examining Figure 5.2 which shows how the game-tree looks when John moves first but his move is hidden from Janet. The dotted line between Janet's decision nodes, Janet₁ and Janet₂, is a simple illustrative device that is used to signify that Janet doesn't know whether she is at Janet₁ or Janet₂.
Janet\textsubscript{2}. Broken lines either joining or around a player’s decision-nodes are used in this way. They indicate that the player with the move doesn’t know which of the nodes joined by or, enclosed within, the broken lines, he or she is at. In this case Janet doesn’t know whether she is at Janet\textsubscript{1} or Janet\textsubscript{2} because she hasn’t seen John’s move.

In the version of the game depicted in Figure 5.2 we cannot use backward induction to analyse Janet’s moves at Janet\textsubscript{1} and Janet\textsubscript{2} because the games beginning at these nodes are not proper subgames of the whole game. Remember from Chapter 4 that the definition of a subgame is a subset of the whole game that starts at a decision-node where there is no uncertainty. As Janet doesn’t see John’s move she doesn’t know whether she is at Janet\textsubscript{1} or Janet\textsubscript{2} so there is clearly some uncertainty for her at each of these decision-nodes. This implies that there are no proper subgames of this version of the game and backward induction cannot be used. Furthermore, as Janet’s moves are not contingent on John’s move – they can’t be as she doesn’t see John’s move – she has only two pure strategies: pub and party. As John moves first there is no possibility of him seeing Janet’s move and therefore he too has only two pure strategies: pub and party. It follows that unless Janet has some information about John’s moves the sequential version of this game with hidden moves is analytically equivalent to the simultaneous-move version. Each player has two pure strategies, neither player has an advantage and there are two Nash equilibria. Since each prefers a different equilibrium neither is more likely than the other.

This indeterminacy is borne out in experiments with battle of the sexes games in which subjects tend to choose randomly and there are many mismatches (Camerer, 2003: 353–67). As you have seen, the theoretical prediction changes if John moves first and Janet sees his move. In experiments this is modelled by allowing one player to announce in advance their intended choice. When this happens the announcing player secures their preferred outcome as predicted. Interestingly, subjects also tend to coordinate on the first mover’s preferred outcome even when the second-mover doesn’t know what the first-mover has done. Camerer (2003: 367) attributes this to ‘a tacit – almost telepathic – first-mover advantage’. Telepathy isn’t assumed in game theory. Instead a first-mover advan-
tage of this kind would be explained by assuming that the second-mover, Janet in Figure 5.2, had some information about the likelihood of John choosing pub. This information would be reflected in her beliefs and these are modelled by probabilities. We have not dealt with probabilities yet but now you are going to see how they can be incorporated into the analysis.

### 5.2 Risk and probabilities

As noted in Chapter 1 not all risky situations are strategic ones and the analysis of risk and uncertainty extends beyond game theory to decision theory more generally (see, for example, Watson and Bude, 1987: Part 1 or Biswas, 1997: Chapters 1–2). In order to keep the analysis simple we can start by considering some situations where there is risk but it is non-strategic. When risk is non-strategic the decision maker has no influence or possible impact on how the risk is resolved.

For example, when you plan your next holiday you will have to make a number of decisions concerning your destination, the timing of your holiday and so forth. You will make these decisions in the knowledge that there is some risk that your plans will be disrupted, perhaps because of a luggage handlers’ strike or your cat being ill, or that the holiday is a disaster for some other reason (for example if the weather is awful). But your choice concerning your holiday doesn’t actually have any effect on any of these possibilities; they will occur or not occur whatever you decide. The luggage handlers, your cat or the weather don’t sit down and think about where or if you are going on holiday and then make a decision about whether to strike, be ill or organise a freak hurricane. The chance of a strike taking place, your cat being ill or the weather being bad is therefore independent of how you actually choose and this means that the luggage handlers, your cat and the weather are not playing a strategic game with you. Other examples of instances where there is non-strategic risk are lotteries, a football match from the standpoint of a supporter and games of chance like roulette. In all of these cases there is only one player, i.e. the decision maker herself, and therefore no strategic interaction of the kind you have been looking at in the previous chapters. People who are superstitious in these circumstances think that the weather, the outcome of the lottery, the match or the spin of the roulette wheel are in some way influenced by what they do. That is, they think they are playing some kind of strategic game but they are not.
The first example of non-strategic risk you are going to look at is the decision faced by Mr Punter on a day out at the races. He is deciding whether or not to bet in the last race of the day on a horse owned by his friend Mr Lucky. Mr Lucky has inside knowledge about his horse and the other horses in the race. He assures Mr Punter that his horse has a 1 in 10 (a 0.1 or $\frac{1}{10}$) chance of winning the race. Mr Punter has €M left in his pocket. If he bets €c on the horse winning the race and the horse wins Mr Punter wins €z making a net gain of €w which we can call €w. If he makes the bet and the horse loses he will take home €M – c. If he doesn’t make the bet he takes home the whole €M. (€M is the amount of money he starts with, c is the cost of the bet and w is his net win if the horse wins.) In making his decision whether to bet on the horse or not Mr Punter knows that there are only two ways in which the uncertainty can be resolved:

- The horse wins (probability 0.1).
- The horse loses (probability 0.9).

However, there are three possible outcomes from Mr Punter’s perspective:

- He bets €c and wins €z so that his total wealth is €M + (z – c) = €M + w.
- He bets and loses €c so that his total wealth is €M – c.
- He doesn’t bet and retains his original monetary wealth €M.

### Exercise 5.1

In the examples (a) to (g) below only the last two could easily be represented as strategic games. Can you say why? (Hint: who are the decision makers in the examples, are there more than one and if so do they both make their decisions by taking into account what they think the other is likely to do?)

(a) Betting on the outcome of a horse race or a cricket match that hasn’t to your knowledge been fixed.
(b) Gambling on a fair roulette wheel at a casino.
(c) Deciding whether to take an umbrella with you when you go out for the day in England.
(d) Deciding whether to take out insurance against the risk of your home being burgled.
(e) A smoker deciding whether to continue smoking or not.
(f) A thug thinking about whether to punch someone in a bar who has annoyed him but who might be a karate expert.
(g) A firm deciding whether to enter a new industrial sector where one firm has a monopoly position but it is unclear whether the incumbent will resist entry or not.
The outcomes, \( \varepsilon M + w \), \( \varepsilon M - c \) and \( \varepsilon M \) are Mr Punter’s contingent pay-offs. They are contingent on Mr Punter’s initial decision and how the uncertainty is resolved. The decision problem for Mr Punter is represented by the diagram in Figure 5.3.

Figure 5.3 is a decision-tree showing the choice problem for Mr Punter. It is not a game-tree as such, as Mr Punter is not involved in a strategic game but the two forms are very similar. Figure 5.3 shows that Mr Punter moves first by deciding to bet on the horse or not. After he has made his decision the race is run and either his horse wins or it doesn’t. As far as Mr Punter is concerned whether his horse wins is decided by chance or nature (a pseudo player) as he has no influence on the outcome. The chance move is depicted by the probabilities written beside the branches of the decision-tree attached to the right of the decision-nodes labelled chance. Chance determines whether the horse wins or not and it wins with probability 0.1 and loses with probability 0.9. The chance move is the same whether Mr Punter bets or not – it is not contingent on his move. Mr Punter's pay-offs are written at the terminal-nodes of the decision-tree. Only his pay-offs are shown as he is the only proper player in the game. The horse, for instance, is not.

Should Mr Punter bet on the horse? Assuming Mr Punter prefers more money to less his decision will depend on the expected pay-off from betting relative to the expected pay-off from not betting. The simplest way to calculate expected pay-offs is to calculate the expected value of the pay-offs from betting and not betting respectively. The expected value of the pay-off from taking a particular decision is the average of the pay-offs associated with all the possible outcomes of that decision. The average is calculated by weighting (or multiplying) each pay-off by the probability that it will occur. The expected value of Mr Punter’s

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**Figure 5.3** Mr Punter’s decision-tree
pay-off from betting is the sum of the weighted pay-offs from winning and losing and each pay-off is weighted by the probability that it occurs. Thus if he bets the expected value of his pay-off is:

- \(0.1(\text{\€} M + w) + 0.9(\text{\€} M - c)\)

This formulation says that the expected value of betting is the probability of winning (0.1) multiplied by the pay-off from betting and winning (\(\text{\€} M + w\)) plus the probability of losing (0.9) multiplied by the pay-off from losing (\(\text{\€} M - c\)). If he doesn’t bet then his wealth is the same whether the horse wins or not therefore his expected pay-off from not betting is:

- \(\text{\€} M\)

That is \(\text{\€} M\) with certainty, a sure thing. If Mr Punter prefers more money to less, then he could decide to bet on the horse if the expected value of betting is greater than the expected value of not betting or:

- \(0.1(\text{\€} M + w) + 0.9(\text{\€} M - c) > \text{\€} M\)

which simplifies to \(\text{\€} w > \text{\€} 9c\).

If Mr Punter chooses to bet because \(w\) is greater than \(9c\) this implies that his utility or satisfaction from the expected pay-off of betting is greater than his utility from the expected pay-off of not betting. We can write Mr Punter’s subjective utility for a given monetary pay-off as \(U(\text{\€})\) where \(U(\text{\€})\) is the function that determines how a given monetary sum translates into units or levels of satisfaction in Mr Punter’s mind. For example, if the monetary sum is \(\text{\€} 100\) then \(U(100)\) is the subjective utility value to Mr Punter of \(\text{\€} 100\). If Mr Punter prefers more money to less then the function \(U(\text{\€})\) will reflect this so that, for example, \(U(100) > U(10) > U(5)\). But the amount by which \(U(100) > U(10)\) will depend on Mr Punter’s preferences which will be unique to him. He may, for example, value \(\text{\€} 100\) more than, less than or exactly ten times as much as \(\text{\€} 10\). His utility function will reflect his preferences in this regard as well (some different possibilities are outlined below).

With this notation the condition that his utility from the expected pay-off of betting is greater than his utility from the expected pay-off of not betting can be written in mathematical shorthand as:

- \(U(0.1(\text{\€} M + w) + 0.9(\text{\€} M - c)) > U(\text{\€} M)\)

where \(U(0.1(\text{\€} M + w) + 0.9(\text{\€} M - c))\) is Mr Punter’s utility from the expected value of betting on the horse and \(U(\text{\€} M)\) is his utility from \(\text{\€} M\) which is the expected value of not betting.

This formulation has the advantage of simplicity in that it implies that if the expected value of one option is greater than another then the decision maker
should simply choose the former. However, because some very risky options
can have the same expected value as other very safe options this formulation
fails to take into account differing attitudes to risk. It implies that all the deci-
sion maker cares about is the overall expected value of a given choice and not
the probabilities that are implicit in that relevant expected value calculation.
For example, it implies that an individual would be indifferent between a sure
payment of €1000 and a lottery ticket with a 1 in 1000 (0.001 or $\frac{1}{1000}$) chance of
winning €1 000 000. This is because if there is a 1 in 1000 chance of winning
with the lottery ticket there is also a 999 in 1000 chance of losing, that is win-
ning nothing. The expected value of the lottery gamble is therefore $(0.001 \times 1$
$000 000 + 0.999 \times 0) = €1000$, the same as the sure payment of €1000. Would
you be indifferent between these two options, or prospects as they are
sometimes called? Even if you are most people would not be. For instance,
some people are risk averse which means that they don’t really like risk and
others, like Formula One racing car drivers and bungee jumpers, appear to love
it and actively seek it out. Alternatively you may not fit into either of these
categories because you neither worry about nor enjoy risk – in this case you are
risk neutral. A further complication is that some people might like to take
small risks, for instance by buying lottery tickets, but they might not be so
keen to take large risks, for example with their lives.

5.2.1 Expected utility

One way of taking these different attitudes to risk into account
is to calculate the expected utility, as opposed to the expected
value, of taking a particular gamble. The expected utility of
accepting a particular gamble (or prospect) is the average utility
derived from the associated contingent pay-offs. It is calculated
by finding the utility of each of the possible contingent pay-offs, weighting
each by the probability that it occurs and then summing to come up with the
overall average. This can be contrasted with the expected value calculation
which probability weights the contingent pay-offs themselves, not the utility
or satisfaction that they potentially bring to the decision maker. These two for-
mulations sound, on the face of it, very similar. But the differences between
them mean that the expected utility of a gamble or prospect will only equal the
utility of the expected value if the decision maker doesn’t care about risk (he or
she is risk neutral) or there is in fact no risk (the gamble is a sure thing). We can
see this by looking at some examples and I’ll start by reconsidering the decision
problem faced by Mr Punter.

As you have already seen the expected value of betting for Mr Punter is
$0.1(M + w) + 0.9(M – c)$. Letting $U(\€)$ define Mr Punter’s utility for money his
utility from the expected value of betting is given by:
● Utility of the expected value of the bet, UEV: \( U(0.1(M + w) + 0.9(M – c)) \).

while the expected utility of betting is determined by probability weighting the utilities of each of the contingent pay-offs \((M + w)\) and \((M – c)\). This leads to:

● Expected utility of the bet, EU: \( 0.1U(M + w) + 0.9U(M – c) \).

This is in effect the expected value of the utility of betting while the utility of the expected value is the utility of the expected monetary value of betting. The two definitions may look and sound very similar but, as noted above, they will only be equal if Mr Punter is risk neutral. Otherwise, by separately probability weighting the utilities of the alternative pay-offs the expected utility formulation will reflect either his aversion to, or love of, risk.

More specifically the theoretical assumptions that underlie the expected utility calculation\(^6\) imply that if he is risk averse the expected utility of betting, EU, will be less than the utility of the expected value, UEV. If he is risk-loving the opposite will be true. More generally if a risky alternative has the same expected value as a sure thing a risk-averse person will prefer the sure thing because the expected utility of the gamble will be less. On the other hand a risk-loving person will prefer the gamble. The expected utility formulation also implies that, if two gambles have the same expected value but one is relatively more risky than the other, a risk-loving person will prefer the more risky gamble, a risk-averse person will prefer the safer gamble and a risk-neutral person will be indifferent between the two prospects. A gamble could be riskier than another that had the same expected value if the probability of losing in the risky gamble is higher but there is also a bigger chance of winning a larger prize.

It follows that Mr Punter (and anyone else who is facing a risky choice) won’t necessarily choose the option with the highest expected value. Instead, if they are either risk-averse or risk-loving\(^7\) they will choose the option with the highest expected utility which may or not be the option with the highest expected value. This doesn’t mean that a risk-averse person will never gamble, just that they will only choose risky options if they have a high enough expected value relative to the expected value of choosing some less risky or risk-free alternative.

To summarise, the expected utility of a gamble is the probability weighted average of the utilities of the pay-offs corresponding to the alternative outcomes that characterise the gamble (the horse winning or losing in Mr Punter’s case). To generalise a little assume that an action has two possible outcomes corresponding to the contingent pay-offs \(x\) and \(y\). The probability of \(x\) occurring is \(p\) and the probability of \(y\) occurring is therefore \((1 – p)\). The expected utility from making the decision to take the action will be:

\[
EU = pU(x) + (1 – p)U(y)
\]

If there is a third possible outcome corresponding to the contingent pay-off \(z\) and the probability of \(z\) occurring is \(q\) then the expected utility of this prospect will be:
If there are more than three possible outcomes then the expected utility formulation is extended accordingly.

### Expected value and expected utility

- **Expected utility, EU**: the expected utility of a gamble is the (probability weighted) average of the utilities of the pay-offs corresponding to the alternative outcomes that characterise the gamble.

- **Expected value, EV**: the expected value of a gamble is the (probability weighted) average of the pay-offs corresponding to the alternative outcomes that characterise the gamble.

- **Utility of expected value, UEV**: the utility of the expected value of a gamble is the utility of the (probability weighted) average of the pay-offs corresponding to the alternative outcomes that characterise the gamble.

### 5.2.2 Expected values, expected utilities and attitudes to risk

To illustrate some of these points consider the gambles A and B described by the following probabilities and prizes.

**Gamble A**

- You win €100,000 with probability 0.01.
- You win nothing with probability 0.99.

**Gamble B**

- You win €2000 with probability 0.5.
- You win nothing with probability 0.5.

Ask yourself which gamble you prefer (assume that entry is costless) and make a note of your answer. The decision-tree corresponding to this choice problem is illustrated in Figure 5.4.
The expected value of gamble A is $0.01 \times 100\,000 + 0.99 \times 0 = 1000$ and the expected value of gamble B is $0.5 \times 2000 + 0.5 \times 0 = 1000$. As the expected values of the two gambles are the same the utilities of the expected values of the two gambles will also be the same. This means that if you don’t attach any value (positive or negative) to risk you should be indifferent between the two gambles. But were you? Possibly not. Gamble B has a higher probability of winning a reasonable prize than gamble A although the prize in gamble A is larger. This means that gamble B is a relatively safer gamble and if, like me, you would prefer gamble B you are, at least in relation to these gambles, risk-averse. If you prefer gambler A you are risk-loving.

With the expected utility formulation we can take account of these different possibilities. To see this, suppose that your utility function, $U(\varepsilon y)$, for an amount of money $y$ is $y^2$ ($U(\varepsilon y) = y^2$). Then the utility you would derive from €10 would be worth 100 (units of utility), the utility you derive from €100 would be 10 000 and so on. With this utility function the utilities of the expected values of the two gambles are:

- Utility of the expected value of gamble A: $UEV(A) = U(0.01 \times 100\,000 + 0.99 \times 0) = U(1000) = 1000^2 = 1\,000\,000$.
- Utility of the expected value of gamble B: $UEV(B) = U(0.5 \times 2000 + 0.5 \times 0) = U(1000) = 1000^2 = 1\,000\,000$.

The utilities of the expected values are the same for both gambles because their expected values are the same. Now let’s look at the expected utility formulations. The expected utility of gamble A is:

- Expected utility of gamble A: $EU(A) = 0.01U(100\,000) + 0.99U(0)$
With $U(€y) = y^2$ we can calculate precisely the expected utility of gamble A as:

- **Expected utility of gamble A:** $EU(A) = 0.01(100000^2)$
  
  \[ = 100000000. \]

and the expected utility of gamble B is:

- **Expected utility of gamble B:** $EU(B) = 0.5U(2000) + 0.5U(0)$
  
  \[ = 0.5(2000)^2 = 2000000. \]

With the utility function $U(€y) = y^2$ the expected utility of gamble A is higher than that of gamble B even though the expected values of the two gambles are the same. An individual with this utility function would therefore choose gamble A (the more risky gamble) in preference to gamble B (the relatively safer gamble). Such a person is risk-loving; he or she prefers a risky option to a safer option with the same expected value. The expected value formulation couldn’t take these kinds of preferences into account but you have seen that the expected utility formulation can.

Now consider the position of Ms Careful whose utility for money is described by the function $U(€y) = \sqrt[3]{y}$ (or $y^{\frac{1}{3}}$). With this utility function the utility Ms Careful derives from €100 is $U(€100) = 10$ and the utility she derives from €4 is $U(€4) = 2$ and so on. The expected values of gambles A and B haven’t changed, both are still 1000, but the utility of these expected values and the expected utilities of the gambles will be different since the utility function is different. As the expected value of gambles A and B are both 1000 Ms Careful’s utility from the expected value of gambles A and B is:

- **Utility of the expected value of gamble A:** $UEV(A) = U(1000) = \sqrt[3]{1000} = 10$
  
  - **Utility of the expected value of gamble B:** $UEV(B) = U(1000) = \sqrt[3]{1000} = 10$

For Ms Careful the expected utility of gamble A is $0.01U(100000) + 0.99U(0)$ and with her utility function this implies:

- **Expected utility of gamble A:** $EU(A) = 0.01 \sqrt[3]{1000000} + 0.99 \sqrt[3]{0}$
  
  \[ = 0.01(316.23) = 3.1623 \]

and the expected utility of gamble B is $0.5U(2000) + 0.5U(0)$ or:

- **Expected utility of gamble B:** $EU(B) = 0.5 \sqrt[3]{2000} + 0.5 \sqrt[3]{0}$
  
  \[ = 0.5(44.72) = 22.36 \]

So for Ms Careful the expected utility of gamble B is higher than that of gamble A and therefore, if she is aiming to maximise her expected utility, she will
choose gamble B, the relatively safer gamble. She is risk-averse since she prefers gamble B, the relatively safe option, to gamble A, the risky option, even though they both have the same expected value.

This exercise has shown that the expected value formulation won’t always reflect people’s different subjective valuations of risky pay-offs. The expected utility formulation, on the other hand, by probability weighting the utilities of the contingent pay-offs can take better account of people’s different attitudes to risk. This is why I said in Section 1.8 of Chapter 1 that the expected utility formulation was potentially a more useful way of calculating a player’s expected pay-off.

The following lottery example shows in a stark way how important attitudes to risk can be. In this example you are offered a choice between two lottery tickets. One is for a midweek lottery and the other is for a Saturday lottery. The prizes and the probabilities of winning in the two lotteries are as follows.

**Midweek lottery**
- 50 per cent chance of winning €10000.
- 50 per cent chance of winning nothing.

**Saturday lottery**
- 100 per cent chance of winning €5000.
The expected values of these two lotteries are the same, €5000, but the midweek lottery is risky – there is a good chance that you will win nothing – while the Saturday lottery is completely safe – you are guaranteed €5000. In these circumstances I would be very surprised if you were indifferent between these two lotteries. Most people would prefer the certainty of the Saturday lottery (which is not really a lottery at all, it is a sure thing). This would imply that they were risk-averse; they prefer a sure thing to a risky prospect with the same expected value. If you prefer the midweek lottery you are risk-loving; you prefer a risky prospect to a sure thing with the same expected value. If you are indifferent between the two lotteries then you are risk neutral. The expected value formulation cannot take account of these different possibilities but the expected utility formulation can.

**Attitudes to risk**

- **Risk love**: risk lovers prefer a gamble to a sure thing with the same expected value.
- **Risk aversion**: risk-averse people prefer a sure thing to a gamble with the same expected value.
- **Risk neutrality**: risk-neutral people are indifferent between a sure thing and a gamble with the same expected value.

If two gambles have the same expected value but one is riskier than the other because there is a higher probability of winning the lowest valued prize but also a higher probability of winning the highest valued prize:

- Risk lovers prefer the riskier gamble.
- Risk-averse people prefer the safer gamble.
- Risk-neutral people are indifferent between the two gambles.

If one gamble has a higher expected value than another then:

- Risk lovers and risk-neutral people will always choose the gamble with the higher expected value.
- Some risk-averse people may choose the gamble with the lower expected value if it appears less risky.
To summarise, if outcomes are uncertain people won’t necessarily choose between prospects according to their expected value. Instead, if people care about risk the choices they make may be more consistent with the expected utility hypothesis. The expected utility formulation can better account for people’s attitudes to risk because the utility of a contingent pay-off (rather than the pay-off itself) is weighted by the probability that it will occur. This means that the overall weighting given to a contingent pay-off depends not just on the probability that it will occur but also on the utility function of the individual concerned. An outcome that has a lower probability of occurring is still given a lower weight but the relative worth of a larger pay-off will be magnified or diminished according to the individual’s utility function.

A person whose utility function is such that he or she attaches more and more incremental (or marginal) value to equal increases in income will be more willing to gamble on higher and higher prizes. Such a person is a risk-lover. Similarly, a person who attaches less and less incremental value to equal increases in income will be less willing to gamble. This kind of person is risk-averse. Someone who values equal gains in income equally is risk-neutral. This implies that the expected utility of a gamble will be less than the utility of the expected value of the gamble if the individual is risk-averse. The opposite will be true if the individual is a risk-lover like Ms Flutter in the boxed example. The two calculations will be equal if the individual is risk neutral.

More formally risk-loving people have increasing marginal utility, risk-averse people have diminishing marginal utility and risk-neutral people have constant marginal utility. This can be made more precise by using some mathematical

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**Exercise 5.2**

You are offered the choice of a free ticket to enter lottery 1 or a free ticket to lottery 2. In lottery 1 and lottery 2 the prizes and the probabilities of winning are as follows:

**Lottery 1**
- 10 per cent chance of winning €5000.
- 70 per cent chance of winning €2500.
- 20 per cent chance of winning nothing.

**Lottery 2**
- 40 per cent chance of winning €5000.
- 20 per cent chance of winning €2500.
- 40 per cent chance of winning nothing.

Which lottery do you prefer? What are the expected values of the tickets for these two lotteries? If you prefer lottery 1 are you risk-averse, risk-neutral or risk-loving in relation to the two lotteries?
terminology. In general, if an individual’s utility function is given by \( U(y) = y^z \), where \( y \) is a monetary (or other objectively valued) pay-off and \( z > 0 \) then if \( z = 1 \) the individual’s marginal utility is constant, it is increasing if \( z > 1 \) and diminishing if \( z < 1 \). Thus the individual is risk-neutral if \( z = 1 \), risk-loving if \( z > 1 \) and risk-averse if \( z < 1 \). The two examples that were used above were \( U(y) = y^2 \) and \( U(y) = \sqrt{y} = y^{1/2} \). In the first case \( z = 2 > 1 \) and in the second \( z = \frac{1}{2} < 1 \). We showed above that with \( U(y) = y^2 \) an individual’s choices would reflect those of a risk-lover and in the second case, that of Ms Careful, her choices were consistent with risk aversion.

The kinds of examples we have looked at show that attitudes to risk are important. In general people don’t like risk and that they commonly take action to avoid risk, for example by buying insurance or diversifying. On the other hand many people who buy insurance also gamble. Often the gambles they take are unfair in that they favour the individual or institution that offers them. This implies they are risk-loving, but buying insurance implies they are also risk-averse. It seems strange that people can be risk-averse and risk-loving at the same time but one explanation is that people have different attitudes to risk in different situations and over different sums of money. If this is true then they will have a utility function that has both risk-averse and risk-loving regions. Alternatively people may gamble for reasons that are unrelated to whether they are risk-loving or not – for example they may enjoy gambling (it is a form of entertainment) or they are mistaken about the element of risk involved (they think the gambles they accept are fair or favour them).

Expected utility theory implies that when individuals choose between risky alternatives they will choose the one with the highest expected utility. This hypothesis is widely used to determine equilibrium strategies in games where there is uncertainty for one or more of the players (you will see examples of this in Chapters 7 and 8). However, the expected utility hypothesis relies on a number of underlying assumptions about the ways in which people make choices between risky prospects and these assumptions have been criticised. For a detailed exposition of these assumptions and a formal derivation of the expected utility hypothesis see Starmer (2000: 334–6) or Camerer (1995: 617–20).

The criticisms levelled at expected utility theory suggest that its applicability maybe limited and as such these limitations should be born in mind when drawing insights from theoretical arguments that rely on the expected utility formulation. The criticisms of the theory fall into two main groups: theoretical and descriptive. Theoretical considerations arise when expected utility theory is unable to capture important elements of an individual’s choice problem. The descriptive limitations follow from experimental evidence of violation of the underlying assumptions of expected utility theory.
5.3.1 Theoretical limitations

The theoretical limitations of expected utility theory include evidence of portfolio effects and issues associated with the temporal resolution of uncertainty.

5.3.1.1 Portfolio effects

Critics argue that the expected utility formulation ignores ‘other gambles’ that are already faced by the individual but may be relevant. Consider the position of Ms Investor who is offered shares in either Marks & Spencer or Manchester United. If she already holds shares elsewhere her choice is likely to be affected by her existing portfolio of holdings. For instance, if she already holds shares in Tottenham Hotspur she may prefer to diversify away from English football. If she prefers to diversify then she will take the Marks & Spencer shares. This consideration implies that the probability distributions of gambles under consideration will not always be sufficient descriptions of the choice problem.

5.3.1.2 Temporal considerations

The expected utility formulation does not take into account the timing of any resolution of uncertainty but this can matter to people. To see this imagine you are offered a choice between €500 for sure or a lottery ticket with a 40 per cent chance of winning a million euros and a 60 per cent chance of winning nothing. A possible drawback of the lottery option is that if you win you won’t get the money until a year from today. However, this is not so much the issue here. What I’d like you to consider is whether your choice between the €500 for sure or the lottery ticket is affected by the timing of the resolution of the uncertainty. By this I mean the time at which you find out whether you have or have not won the lottery. Consider the following possibilities:

(a) The lottery is drawn today and you find out whether you have won or not today.
(b) The lottery is drawn a year from today and you find out whether you have won or not a year from today.
(c) The lottery is drawn today and you find out whether you have won or not a year from today.

If you, like me and many others, would prefer (a) and you are indifferent between (b) and (c) then getting the information sooner is valuable to you. This could be because you can only plan your life properly if you have the relevant information or it may be because you just don’t like not knowing. Whatever the explanation, uncertainty has a negative value for you if you prefer (a). Furthermore indifference between (b) and (c) implies that the important time is when the uncertainty is resolved for you, not when the uncertainty...
Limitations of expected utility theory

is ‘physically’ resolved. Expected utility theory is unable to handle these kinds of issues. As Kreps (1990: 114) says, the problem is one ‘of a partial or incomplete model’ of the choice problem. Furthermore, people in general don’t like uncertainty and the expected utility formulation cannot really take this possibility into account. Of course you may not prefer (a). Maybe you like uncertainty? Perhaps it makes your life more exciting? But expected utility theory cannot incorporate preferences like these either.

5.3.2 Descriptive limitations

The logic of expected utility theory follows from a number of underlying assumptions or axioms (see Machina, 1989: 17 or Camerer, 1995: 618–19 for a summary). But experimental evidence suggests that people’s behaviour is not always consistent with two of these: the independence axiom and the axiom of transitivity. This evidence weakens the descriptive claims of expected utility theory, that is its claims to be able to accurately predict what people will actually do when confronted with risk. However, this doesn’t automatically mean that the prescriptive or normative claims of the theory are similarly weakened; we might still want to recommend that people act in accordance with the theory even if they don’t always conform to the theory’s predictions in practice. Nevertheless the evidence that people do not always act in ways that are consistent with the axioms of independence and transitivity represents a serious attack on the validity of expected utility. We are going to examine some of that evidence here first in relation to the independence axiom and secondly in relation to the axiom of transitivity. For a more detailed discussion you could look at Kahneman (2003), Starmer (2000), Camerer (1995) or Machina (1987 or 1989).

5.3.2.1 The independence axiom

The independence axiom of expected utility theory says that the choice between risky gambles or prospects should not be affected by elements that are the same in each. Consider gambles A and B below where x, y, z, v and w are monetary prizes:

- Gamble A: 0.5x + 0.25y + 0.25z
- Gamble B: 0.5v + 0.25w +0.25z

Gamble A has a 0.5 probability of winning the prize x, a 0.25 probability of winning the prize y and a 0.25 probability of winning the prize z. Gamble B offers a 0.5 probability of winning the prize v, a 0.25 probability of winning w and, just as in gamble A, a 0.25 chance of winning z. Thus the gambles have a common element or consequence, z, and a common probability of winning it. The independence axiom claims that your choice between gambles A and B should be independent of this common consequence and the probability of
winning it and therefore your choice should not be affected by either the probability of winning \( z \) or the value of \( z \). This does not seem unreasonable but in experiments where individuals are asked to choose between risky prospects behaviour consistent with the independence axiom is not consistently observed.

Before we consider this experimental evidence consider the gambles on offer in choice problems A and B below. In each case you should make a choice between the gambles you are offered. Make a note of the gambles you prefer in A and B and we will return to them later.

**Choice problem A.** \( a \) and \( b \) are two gambles. Gamble \( a \) has a certain pay-off of \( €1,000,000 \) and gamble \( b \) will give you a 10 per cent chance of winning \( €5,000,000 \), an 89 per cent chance of winning \( €1,000,000 \) and a 1 per cent chance of winning nothing. **Which gamble do you prefer, \( a \) or \( b \)?**

**Choice problem B.** \( e \) and \( f \) are two gambles. Gamble \( e \) will give you \( €700 \) with certainty. Gamble \( f \) will give you an 80 per cent chance of winning \( €1,100 \) and a 20 per cent chance of winning nothing. **Which gamble do you prefer, \( e \) or \( f \)?**

Evidence of violations of the independence axiom is commonly divided into two effects known as (i) common consequence and (ii) common ratio effects.

**(i) Common consequence effects**

Consider an illustrative choice problem between two options \( S \) and \( R \) where:

- **Option S:** \( p_b + q_b + r_a + s_b \)
- **Option R:** \( p_a + q_c + r_a + s_b \)

\( a, b, \) and \( c \) are three monetary prizes such that \( a < b < c \) and \( a = 0 \). \( p, q, r, s \) are the probabilities of winning in \( S \) and \( R \). If \( r > 0 \) then \( s = 0 \) and if \( s > 0 \) then \( r = 0 \). The common consequence in the two options is therefore \( a \) if \( s = 0 \) and \( b \) if \( r = 0 \). As \( a = 0 \) the gambles are both riskier when \( s = 0 \) and \( r > 0 \).

Thus if \( r > 0 \) (in which case \( s = 0 \)) by choosing \( S \) you have a \( (p + q) \) chance of winning \( b \), and an \( r \) chance of winning nothing, that is \( a \). If you choose \( R \) you have a \( (p + r) \) chance of winning nothing, i.e. \( a \), but a \( q \) chance of winning the bigger prize of \( c \). If \( s > 0 \) (in which case \( r = 0 \)) then \( S \) represents a sure thing, a certain prize of \( b \), while \( R \) is a \( p \) chance of nothing, a \( q \) chance of \( c \) and an \( s \) chance of \( b \). In either case \( S \) is the relatively safer option because with \( S \) there is less chance of loosing (winning \( a \)). But the biggest prize, \( c \), can only be won by choosing \( R \). The choice between \( S \) and \( R \) is illustrated in Matrix 5.2 which shows the pay-offs and probabilities corresponding to the two options. This type of matrix is sometimes called a state contingent matrix.
The independence axiom implies that an individual’s choice between S and R should be independent of r and s and therefore independent of whether \( r = 0 \) or \( s = 0 \). Letting \( r = 0 \) or \( s = 0 \) simply amounts to scaling up or down the common consequence and should have no affect on people’s choices. But when gambles like S and R have been presented to people in experiments researchers have found that there is a systematic tendency for them to (i) prefer S when \( r = 0 \) and the common consequence is greater than zero and (ii) prefer R when \( s = 0 \) and the common consequence is zero. Such behaviour violates the independence axiom because it implies that people’s choices are affected by the value of common consequences. Evidence of the common consequence effect was first discovered by Maurice Allais (1953) and was originally referred to as the Allais paradox. Before examining in detail the example Allais used to illustrate this paradox consider the gambles in choice problem A* below. Make a choice between the gambles you are offered and make a note of your choice.

Choice problem A*. c and d are two gambles. Gamble c will give you an 11 per cent chance of winning €1000000 and an 89 per cent chance of winning nothing. Gamble d will give you a 10 per cent chance of winning €500000 and a 90 per cent chance of winning nothing. Which gamble do you prefer, c or d?

Allais’ example is represented here by attaching specific values to the prizes and probabilities in the S and R gambles in Matrix 5.2 as shown in Matrices 5.2.1 and 5.2.2. In Allias’ example S and R are offered in two different situations. In situation 1 \( r = 0 \) and \( s = 0.89 \) and in situation 2 \( r = 0.89 \) and \( s = 0 \). As shown in Matrix 5.2.1 in situation 1 the choice between S and R is between 1 million for sure if you choose S and a 0.89 chance of 1 million, and a 0.1 chance of 5 million and a 0.01 chance of nothing if you choose R. In situation 2 represented in Matrix 5.2.2 S offers a 0.11 chance of 1 million and a 0.89 chance of nothing while R is a 0.1 chance of 5 million and a 0.9 chance of nothing.
You should ask yourself whether you prefer S or R in each case. In situation 1 S is a safe option, a sure thing and R is relatively risky although there is a chance of winning 5 million. After choosing between S and R in situation 1 ask yourself whether you would make the same choice in situation 2? Again S is safer than R but S is no longer a sure thing.

According to the independence axiom your choice between R and S in situation 1 should be independent of the 0.89 chance of winning 1 million as this is a common consequence. Similarly your choice in situation 2 should be independent of the 0.89 chance of winning nothing. As S and R are otherwise identical, the independence axiom implies that if you choose S when r = 0 in situation 1 then you should also choose S in situation 2 when s = 0. Did you conform to this prediction? Maybe you did, maybe not. You can check again by looking at your choices in response to A and A*. The choice problems A and A* are in fact the same as those represented in Matrices 5.2.1 and 5.2.2 (see Machina, 1989: 22). Gambles a in A and c in A* correspond to the S options in situations 1 and 2 respectively while gambles b and d correspond to the R options in situations 1 and 2. Choices consistent with expected utility theory and the independence axiom are either a in A and c in A* or b in A and d in A*.

If you chose b in A and c in A* or a in A but d in A* then you violated the independence axiom.

If your choices were not consistent with the independence axiom don’t worry, you are in good company. There is in fact considerable evidence that people systematically prefer S in situation 1 (or a in A) when r = 0 but they prefer R in situation 2 (i.e. d in A*) when s = 0. Similar violations of the independence axiom have been found to occur in other examples where r and s are large relative to q. In the Allais example when r = 0, S is totally safe and R is
relatively risky but when \( s = 0 \) both \( S \) and \( R \) are risky and \( S \) is only marginally safer than \( R \). In these circumstances, choices in violation of the independence axiom can be defended on the grounds that when \( r = 0 \), \( S \) is clearly the safest option but when \( s = 0 \), as both options are very risky and \( R \) is only marginally more risky than \( S \), it’s worth having a gamble on \( R \).

Evidence of behaviour that violates the independence axiom weakens the descriptive claims of expected utility theory. However, the normative claims of the theory are also weakened if people continue to violate the independence axiom after the axiom has been explained to them and its logic justified. Unfortunately, for expected utility theory there is evidence that people do just that. That is, they continue to violate the independence axiom even after it has been explained to them (see Slovic and Tversky, 1974). In fact anyone who read the first part of this section and then violated the independence axiom in their responses to \( A \) and \( A^* \) also did so after the independence axiom had been explained to them.

**(ii) The common ratio effect**

The independence axiom implies that choices should also be independent of the probability of a common consequence but there is experimental evidence that contradicts this prediction. Consider options \( S' \) and \( R' \) where:

- Option \( S' = \lambda P x_2 + (1 - \lambda) P x_2 + (1 - P) c \)
- Option \( R' = \lambda P x_3 + (1 - \lambda) P x_1 + (1 - P) c \)

In \( S' \) and \( R' \), \( x_1, x_2 \) and \( x_3 \) are prizes, \( c = x_1 < x_2 < x_3 \) and often in experiments \( c = x_1 = 0 \). \( \lambda P \), \((1 - \lambda) P \) and \((1 - P)\) are the probabilities of winning the prizes where \( 0 < \lambda < 1 \), \( 0 < P \leq 1 \). The common ratio in question is the ratio of the probabilities of winning \( x_2 \) or \( x_3 \) in \( S' \) and \( R' \), namely:

\[
\frac{\text{Prob. of winning in } S'}{\text{Prob. of winning in } R'} = \frac{P}{\lambda P} = \frac{1}{\lambda}
\]

The independence axiom implies that choices between \( S' \) and \( R' \) should be independent of \((1 - P)\) and therefore \( P \). However, there is evidence that people systematically prefer \( S' \) when \( P \) is high and prefer \( R' \) when \( P \) is low.\(^{12} \)

Before examining a numerical example consider the choice problem in \( B^* \) below. Without looking back at choice problem \( B \), make a choice between \( g \) and \( h \) and note which gamble you prefer:

**Choice problem \( B^* \).** \( g \) and \( h \) are two gambles. Gamble \( g \) will give you a 20 per cent chance of winning €700 and an 80 per cent chance of winning nothing. Gamble \( h \) will give you a 16 per cent chance of winning €1100 and an 84 per cent chance of winning nothing. **Which gamble do you prefer, \( g \) or \( h \)**?
The choice problems in B and B’ can be written in the same way as options S’ and R’. In B and B’ λ = 0.8, c = x₁ = 0, x₂ = €700, x₃ = €1100. But P = 1 in choice problem B and P = 0.2 in choice problem B’. The common ratio is 1.25 (i.e. \( \frac{1}{0.8} \)) when P = 1 in B and \( \frac{0.2}{0.16} \) when P = 0.2 in B’. Gambles e and f in B correspond to S’ and R’ and are defined as follows:

- Option S’ = gamble e = 0.8€700 + 0.2€700 = €700 for sure.
- Option R’ = gamble f = 0.8€1100 + 0.2€0 = 0.8€1100, i.e. a 0.8 or an 80 per cent chance of winning €1100 and a 20 per cent chance of nothing.

Thus when P = 1 e is the safe gamble corresponding to option S’ and f is the risky gamble corresponding to option R’. In choice problem B’ P = 0.2 and gambles g and h correspond to S’ and R’ and are given by:

- Option S’ = gamble g = (0.8 × 0.2)€700 + (0.2 × 0.2)€700 + 0.8€0 = 0.2€700 i.e. a 0.2 or 20 per cent chance of winning €700 and an 80 per cent chance of nothing.
- Option R’ = gamble h = (0.8 × 0.2)€1100 + (0.2 × 0.2)€0 + 0.8€0 = 0.16€1100, i.e. a 0.16 or 16 per cent chance of 1100 and an 84 per cent chance of nothing.

Thus when P = 0.2 g is the safe gamble corresponding to option S’ and gamble h is the risky gamble corresponding to option R’.

The independence axiom implies that your choice between gambles e and f and gambles g and h should be independent of the value of P. But gambles e and g and gambles f and h are the same other than that P = 1 in gambles e and f and P = 0.2 in gambles g and h. Consequently, the independence axiom implies that if you chose gamble e when P = 1 in B then that you should have chosen gamble g in B’ when P = 0.2. Similarly if you prefer gamble f when P = 1 in B you should prefer gamble h in B’ when P = 0.2. Were your choices consistent with the independence axiom? Maybe they were but once again you would not be alone if they were not. Experiments have shown that in examples like this people have a systematic tendency to choose option S’ when P = 1 and S’ is a sure thing but to choose option R’ when P < 1. Systematic evidence of common consequence and common ratio effects in experiments suggests that the independence axiom does not always reflect what people do. This evidence weakens the claims of expected utility theory to be a descriptive theory. These claims are further weakened by evidence of violations of the transitivity axiom.

### 5.3.2.2 Transitivity

The axiom of transitivity implies that if A is preferred to B and B is preferred to C then A will be preferred to C. This claim appears entirely uncontroversial. For example, if you prefer chocolate to beer and beer to apple pie then you should
prefer chocolate to apple pie. But it seems that when choices are over risky prospects there are instances where the axiom of transitivity is systematically violated. Examples of behaviour that contradict transitivity are usually referred to as instances of preference reversal. Before examining a generalised problem consider the specific choices in problem C below and make a note of your answers to (i) and (ii).

**Choice problem** C. P and $ are two gambles. Gamble P will give you an 80 per cent chance of winning $100 and a 20 per cent chance of winning nothing. Gamble $ will give you a 40 per cent chance of winning $500 and a 60 per cent chance of winning nothing.

(i) Which gamble do you prefer, gamble P or gamble $?
(ii) How much would you be willing to pay to take part in each of the gambles P and $?

Now consider the following example characterised by two gambles or bets: a P bet and a $ bet. The P bet and the $ bet are defined as follows:

- The P bet: a p chance of winning X and a (1 – p) chance of winning x.
- The $ bet: a q chance of winning Y and a (1 – q) chance of winning y.

Where X, x, Y and y are prizes, p and q are probabilities and X > x, Y > y, Y > X and p > q.

The P bet offers a higher chance of winning the smaller prize, X, but the $ bet offers a smaller chance of winning the bigger prize, Y. The choice problem in C is a numerical example of this kind. In C, X = $100, Y = $500, x = y = 0, p = 0.8 and q = 0.4. In experiments involving similar P and $ bets subjects are asked to choose between the P bet and $ bet and to value the P and $ bets (as you were asked to do in C). In effect both tasks amount to answering the same question, namely which of these two bets do you prefer? The axiom of transitivity implies that whichever bet you prefer (in your answer to part (i)) you should assign a higher value to it (in your answer to part (ii)). However, researchers have found a systematic tendency for people to choose the P bet in these circumstances and put a higher value on the $ bet. This has been interpreted in terms of intransitive preferences or preference reversal.15

To see this we can consider separately the implications of placing a higher value on the $ bet but stating a preference for P (see Machina, 1989: 32). If a subject’s valuation of the P bet is $E_p$ and their valuation of the $ bet is $E_\$, then the subject is:

(a) indifferent between the P bet and some sure or certain amount $CE_p$ implying that $P Indiff CE_p$, where Indiff is shorthand for saying that the individual is indifferent between P and CE_p.
(b) Indifferent between the $ bet and some sure value $CE_s$ implying that $ I n d f \ CE_s$.

And if the subject puts a higher value on the $ bet this implies:

(c) $CE_s$ is greater than $CE_p$ which if the individual concerned prefers more money to less implies that he or she prefers $CE_s$ to $CE_p$ or $CE_s \ PT \ CE_p$ where $PT$ is shorthand for preferred to.

However, if the subject chooses $P$ over $\$ then they:

(d) strictly prefer $P$ to $\$. That is, $P$ is preferred to $\$ or $P \ PT \ $. 

As (a) says that $P \ Indf \ CE_p$ and (b) says that $\ Indf \ CE_s$ then if $P \ PT \ $ it must also follow that $CE_p \ PT \ CE_s$. But (c) says that $CE_s \ PT \ CE_p$ which is a contradiction and (a)–(d) together imply that:

(e) $CE_p \ Indf \ P \ PT \ $ or $CE_s \ PT \ CE_p \ Indf \ P$.

(e) simultaneously implies that $P$ is preferred to $\$, $P \ PT \ $ and $\$ is preferred to $P$, $\ PT \ P$ which is inconsistent with transitivity and instead implies intransitive preferences or preference reversal. An alternative interpretation favoured by psychologists is in terms of response mode or framing effects. This interpretation suggests that people respond differently to analytically equivalent questions that are framed differently or have different reference points. For instance, in the P and $ bet example the same question is asked in terms of first choice and then valuation and it is conceivable that choice and valuation invoke different mental responses. Framing effects of this kind have been observed in other experiments where different aspects of the same problem are highlighted, for example when subjects are asked to make decisions in relation to monetary losses or gains or survival and mortality (Kahneman and Tversky, 1979; Tversky and Kahneman, 1981; and see Kahneman (2003) for a recent review of framing effects).

Experimental evidence of preference reversal and violations of the independence axiom clearly weaken the descriptive claims of expected utility theory but to what extent? How meaningful are these experimental results which are after all conducted in a laboratory? Some of the experiments conducted relate to very specific cases, for example where the probability of the common consequence is very large as in Allais’ famous example. Nevertheless the evidence is difficult to ignore and it should be no surprise that a number of alternative theories of choice under risk have been developed such as prospect theory (Kahneman and Tversky, 1979; Kahneman, 2003), generalised expected utility theory (Machina, 1982) and regret theory (Loomes and Sugden, 1986).
In Section 5.1 of this chapter extensive forms were used to model uncertainty about where a player is in a game. A simple device of a broken line linking decision-nodes was used to illustrate uncertainty for players about where they are in a game. This technique was used to show that hidden-move games can be analysed as simultaneous-move games.

In Section 5.2 the idea of non-strategic risk was considered. Risk is incorporated in decision-making problems by specifying the likelihood of different events in terms of probabilities. Individuals are then assumed to choose between gambles or prospects on the basis of their expected pay-offs. Two alternative ways of calculating an expected pay-off were considered: expected value and expected utility. Examples were used to show that only the expected utility formulation can incorporate individuals’ attitudes to risk. Individuals may be risk-averse, risk-neutral or risk-loving. Since not everyone is risk-neutral the expected utility formulation is potentially more useful when analysing decision problems characterised by risk. This conclusion will also apply to strategic games. If players’ pay-offs in games are formulated in terms of their utility then the calculation of an expected pay-off will automatically generate an expected utility. An expected pay-off calculated in this way will take into account a player’s attitude to risk. If the players’ pay-offs are written in terms of some objective measure, such as units of money, they won’t do this since the calculation of the expected pay-off will only yield an expected value.

Expected utility theory is not without its critics and in Section 5.3 some of the limitations of expected utility theory were considered. Experimental evidence of violations of the underlying assumptions of expected utility theory weakens the descriptive claims of the theory. As you will see in subsequent chapters expected utility theory is fundamental to game theoretic analysis when information is incomplete. It follows that experimental evidence of the kind discussed in Section 5.3 must also weaken the descriptive claims of game theory. A question remains as to why people don’t automatically conform to the rules proscribed by expected utility theory. Perhaps in some instances they are just mistaken or ill informed and in such cases the normative or prescriptive claims of expected utility theory (and game theory by assumption) are less affected. However, you should be aware that experimental evidence has raised some challenging questions about the validity of expected utility in particular and game theory more generally. These questions have not been resolved although behavioural game theorists are making inroads in this area (see, for example, Kahneman, 2003 or Camerer, 2003).
5.1
In the examples given only the last two could easily be represented as strategic games. In examples (a) to (e) the risk faced by the decision maker is independent of the choice made. For example, in (b) the outcome of the spin of the roulette wheel is unaffected by a punter’s decision to bet or not (what happens, happens). But in example (f) if the thug decides to punch the other man this will clearly affect him and therefore the other man may wish to take some sort of preventative action, whether he is a karate expert or not (a game like this is analysed in Chapter 7). In example (g) the potential entrant needs to think about the likely response to entry of the incumbent firm and the latter’s profits will depend not only on whether there is entry or not but also on what the incumbent does if there is entry. This is a classic game theoretic scenario and you saw it modelled in Chapter 4. You will see it again in Chapters 7 and 8 where the extra element of uncertainty suggested here is considered.

5.2
You were offered the choice of a free ticket to enter lottery 1 or a free ticket to lottery 2. In lottery 1 and lottery 2 the prizes and the probabilities of winning are as follows:

**Lottery 1:**
- 10 per cent chance of winning €5000.
- 70 per cent chance of winning €2500.
- 20 per cent chance of winning nothing.

**Lottery 2:**
- 40 per cent chance of winning €5000.
- 20 per cent chance of winning €2500.
- 40 per cent chance of winning nothing.

The expected values of lottery 1 is:

- $EV(1) = 0.1\times5000 + 0.7\times2500 + 0.2\times0 = 500 + 1750 = 2250$.

The expected value of lottery 2 is:

- $EV(2) = 0.4\times5000 + 0.2\times2500 + 0.4\times0 = 2000 + 500 = 2500$. 
The expected value of lottery 2 is higher but lottery 2 is riskier; there is greater chance of winning nothing even though there is a higher probability of winning the larger prize. If you prefer lottery 1 you must be risk-averse. If you are risk-loving or you are risk-neutral you will choose lottery 2.

1 What is the expected value of a gamble where you toss a coin and win €100 if it lands heads and lose €50 if it lands tails?

2 A bloke in an English pub is trying to exchange his Scottish pound notes for English pound notes. Legally a Scottish pound note is worth exactly the same as an English pound note. However, there is 10 per cent chance that the Scottish notes are forgeries. If I am risk-neutral how much should I be willing to pay for a Scottish pound note?

Questions 3 to 7 refer to the following scenario: Mr X is aiming to maximise his expected utility (he is an expected utility maximiser). His utility for money is given by the function $U(y) = y^{1/2}$ where $y$ is a monetary pay-off. Mr X’s total income is €10,000. He is offered a bet on the outcome of the toss of a fair coin where, if the coin comes up heads, he loses all his income, but if it comes up tails he doubles it (by winning an extra €10,000).

3 What is Mr X’s expected utility if he takes the bet?

4 What is Mr X’s expected utility if he rejects the bet?

5 Will he take the bet?

6 Mr X is now offered a bet where if a fair coin comes up heads he again loses everything, but if it comes up tails he wins €40,000, taking his total income to €50,000. Will he take the bet?

7 Is Mr X risk-averse, risk-loving or risk-neutral?

Questions for discussion

1 How well does expected utility theory hold up as either (a) a descriptive theory or (b) a prescriptive (or normative) theory in the light of experimental evidence contradicting the independence and transitivity axioms?
Consider the following choice problem attributable to Tversky and Kahneman (1981):

You have to make a decision for yourself or a close friend between surgery or radiation therapy. You are given the following information:

Surgery: of 100 people having surgery 90 live through the post-operative period, 68 are alive at the end of the first year and 34 are alive at the end of 5 years.

Radiation therapy: of 100 people having radiation therapy all live through the treatment, 77 are alive at the end of one year and 22 at the end of 5 years.

Which treatment would you choose? Would you choose differently if the problem was phrased as follows?

You have to make a decision for yourself or a close friend between surgery or radiation therapy. You are given the following information:

Surgery: of 100 people having surgery 10 die during surgery or the post-operative period, 32 die by the end of the first year and 66 die by the end of 5 years.

Radiation therapy: of 100 people having radiation therapy none die during treatment, 23 die by the end of one year and 78 die by the end of 5 years.

If your choice changes why do you think this might be? If your choice doesn’t change can you explain why in experiments researchers have observed a systematic tendency for less people to choose radiation when the problem is phrased in terms of survival than when it is phrased in terms of mortality?

Answers to problems

1 The expected value of the gamble is \( \frac{1}{2}(\text{\€100}) - \frac{1}{2}(\text{\€50}) = \text{\€25}. \)

2 The expected value of a Scottish note is 0.9 English pounds (90 pence). I should be willing to pay 90 pence for each note.

3 Mr X’s expected utility if he takes the bet is \( \frac{1}{2}0 + \frac{1}{2}(20000^{1/2}) = 70.71. \)

4 Mr X’s expected utility if he doesn’t take the bet is \( 10000^{1/2} = 100. \)

5 Mr X will not take the bet. His expected utility if he doesn’t take the bet is 100. If he takes the bet his expected utility is only 70.71.

6 His expected utility if he takes the new bet is \( \frac{1}{2} (50000^{1/2}) = 111.8. \) If he doesn’t take the bet his expected utility is still 100. Since 111.8 > 100 he will take the new bet.
7 He is risk-averse. His expected utility from any gamble is less than the utility of its expected value. Alternatively, you could show that the second derivative of the expected utility function (over certain money) is negative.

1 The probabilities in Mr Punter’s decision problem are objectively given to him by the horse’s owner. When probabilities are determined in this way the situation is said to be one of risk. An alternative possibility is that Mr Punter has no inside knowledge and instead bases his decision on his own subjective assessment of the horse’s chance of winning the race. In this case the probabilities in Mr Punter’s decision problem would be subjectively determined and technically speaking this would imply that the situation was one of uncertainty rather than risk (see Starmer, 2000: 334 for an exact distinction between these two terms).

2 These alternative possibilities are sometimes referred to as states of nature or the world. This terminology is more usual when the decision-making problem is characterised by uncertainty rather than risk (see previous note).

3 Starmer (2000) defines a prospect as a list of consequences with associated probabilities.

4 If you want to know more about attitudes to risk, expected values and expected utilities you could start by consulting the relevant chapter in a microeconomics textbook for example Katz and Rosen (1998: Chapter 6, pp. 167–8) or Frank (2003: Chapter 6, pp. 212–15).

5 Expected utilities are often referred to as von Neumann-Morgenstern utility functions after the originators of expected utility theory. For a precise formulation see Starmer (2000). For a less formal introduction consult a microeconomics textbook such as Katz and Rosen (1998: Chapter 6, Section 4).

6 For a detailed exposition of these assumptions and a formal derivation of the expected utility hypothesis see Starmer (2000: 334–6).

7 That is they care about risk and by assumption are choosing to maximise expected utility rather than expected value.

8 The individual’s marginal utility is given by differentiation as \( zy^{z-1} \) and the rate of increase (or decrease) in marginal utility is \( (z - 1)zy^{z-2} \). The latter will equal zero if \( z = 1 \) implying constant marginal utility, it is positive if \( z > 1 \) implying increasing marginal utility and it is negative if \( z < 1 \) implying diminishing marginal utility.

9 Fair gambles have a zero expected value, for example a game where if a coin comes down heads you win €10 and you lose €10 otherwise. The expected value of this game is \( \frac{1}{2}10 - \frac{1}{2}10 = 0 \). The expected value of your wealth if you accept this gamble is the same as the certain value of your wealth if you refuse the gamble. For example, if your initial wealth is €40 then the expected value of your wealth if you take the gamble is \( \frac{1}{2}(40 + 10) + \frac{1}{2}(40 - 10) = 40 \). By definition risk-averse people always refuse fair gambles, risk-neutral people are indifferent about fair gambles and risk-lovers always accept fair gambles (see Frank, 2003: Chapter 6 for a more detailed discussion). The gambles offered by the gambling industry must be unfair in this technical sense or the industry could not expect to make profits.

10 See, for example, Kemp (1988).

11 See, Kreps (1990, Chapter 3: Section 5) for a more detailed discussion of these issues.

12 See, for example, Starmer and Sugden (1989), Machina (1989: 25–6) or Camerer (1995: 623–4) for further discussion.
13 If your choices were consistent with the independence axiom would they still be if the prizes were in tens of thousands of euros?

14 Apparently, it is not just humans who violate the independence axiom. Battalio, Kagel and MacDonald (1985) found that in experiments with rats some of their non-human subjects exhibited common ratio effects.

15 This pattern of violation was first observed by Lichentenstein and Slovic (1971) and Lindman (1971). See Tversky and Thaler (1990) for a review of the related evidence.

16 CEp is called the certainty equivalent of P.