GAME THEORY TOOLBOX

Concepts and techniques

- Strategic interdependence
- Players
- Strategies
- Pay-offs
- Utility
- Equilibrium
- Simultaneous-move games, static games
- Strategic form, pay-off matrix
- Sequential-move games, dynamic games
- Extensive form, game tree
- Repeated games
- Constant-sum and zero-sum games
- Cooperative games.

After working through this chapter you will be able to:

- Describe a strategic situation as a game
- Explain the difference between simultaneous moves and sequential moves in games
This chapter sets out a framework for understanding and applying game theory. It provides you with the tools that will enable you to use game theory to analyse a range of different problems. The general approach of game theory is outlined in the first part of the chapter; what it is and how and when it can be used. You will also see some examples of situations that could be usefully analysed as games. Some of the everyday language used by game theorists is explained and the type of outcome predicted by game theory is characterised. Two main categories of games are simultaneous-move games and sequential-move or dynamic games. These are both described in this chapter. You will see how pay-off matrices are used to capture the salient features of simultaneous-move games and how extensive forms or game trees are used to illustrate dynamic games. Games can be played only once or repeated, they can be co-operative or non-co-operative. Sometimes the participants in a game have shared interests and sometimes they don’t. These distinctions are all explained. In some games the participants will have the same information and in others they won’t. The amount of information in a game can affect its outcome and this possibility is discussed in the last section of this chapter. In the subsequent chapters of this book, the terminology that you are introduced to in this chapter and the different approaches that are outlined, will be developed so that you use game theory to interpret, explain and make predictions about the likely outcomes of decision problems that can be analysed as games.
The first important text in game theory was *Theory of Games and Economic Behaviour* by the mathematicians John von Neumann and Oskar Morgenstern published in 1944.¹ Game theory has evolved considerably since the publication of von Neuman and Morgenstern’s book and its reach has extended far beyond the confines of mathematics. This is due in a large part to contributions in the 1950s from John Nash (1950, 1951). However, it was in the 1970s that game theory as a way of analysing strategic situations began to be applied in all sorts of diverse areas including economics, politics, international relations, business and biology. A number of important publications precipitated this breakthrough, however, and Thomas Schelling’s book *The Strategy of Conflict* (1960) still stands out from a social science perspective.

Hutton (1996: 249) describes game theory as ‘an intellectual framework for examining what various parties to a decision should do given their possession of inadequate information and different objectives’. This definition describes what game theory can be used for rather than what it is. It also implicitly characterises the distinctive features of a situation that make it amenable to analysis using game theory. These features are that the actions of the parties concerned impact on each other but exactly how this might happen is unknown. Interdependence and information are therefore critical aspects of the definition of game theory.

Game theory is a technique used to analyse situations where for two or more individuals (or institutions) the outcome of an action by one of them depends not only on the particular action taken by that individual but also on the actions taken by the other (or others). In these circumstances the plans or strategies of the individuals concerned will be dependent on expectations about what the others are doing. Thus individuals in these kinds of situations are not making decisions in isolation, instead their decision making is interdependently related. This is called *strategic interdependence* and such situations are commonly known as *games of strategy*, or simply games, while the participants in such games are referred to as *players*. In strategic games the actions of one individual or group impact on others and, crucially, the individuals involved are aware of this.

Because players in a game are conscious that the outcomes of their actions are affected by and affect others they need to take into account the possible actions of these other individuals when they themselves make decisions. However, when individuals have limited information about other individuals’ planned actions (their *strategies*), they have to make conjectures about what they think they will do. These kinds of thought processes constitute strategic thinking and when this kind of thinking is involved game theory can help us to understand what is going on and make predictions about likely outcomes.²
Strategic thinking characterises many human interactions. Here are some examples:

(a) Two firms with large market shares in a particular industry making decisions with respect to price and output.

(b) Leaders of two countries contemplating a war with each other.

(c) The decision by a firm to enter a new market where there is a risk that the existing or incumbent firms will try to fight entry.

(d) Economic policy makers in a country contemplating whether to impose a tariff on imports.

(e) Leaders of two opposing factions in a civil war who are attempting to negotiate a peace treaty.

(f) Players taking/facing a penalty in association football.

(g) A tennis player deciding where to place a serve.

(h) Managers involved in the sale and purchase of players on the transfer market in association football.

(i) A criminal deciding whether to confess or not to a crime that he has committed with an accomplice who is also being questioned by the police.

(j) The decision by a team captain to declare in cricket.

(k) Family members arguing over the division of work within the household.

In all of the above situations the participants or players are involved in a strategic game. The outcome of their planned actions depends on the actions of others players and therefore their plans may be thwarted in that they do not achieve their desired outcome. For example, in scenario (a) the players are firms with large market shares. Markets where a small number of large firms control a
large share of the market are called oligopolies. An example of an oligopoly is the automobile industry which is dominated by a small number of large multinational companies all of whom are household names (the top five in terms of sales are General Motors, Ford, Daimler Chrysler, Toyota and Volkswagen). Because the firms in an oligopoly are large relative to the size of the industry as a whole, the actions of the firms are independent. For instance, if one firm lowers its price the others are likely to lose custom to the price cutter, or if one firm raises its output by any significant amount the market price will probably fall. In both instances, the profits of the other firms will be lower because of the action of the first firm.

**Exercise 1.1**

In examples (b) to (k) above can you identify the players and explain why and how their actions are interdependent?

There are no wrong or necessarily right answers to Exercise 1.1 but just by thinking about examples like these you will be thinking about strategic situations. This means you will already be starting to think strategically.

Strategic thinking involves thinking about your interactions with others who are doing similar thinking at the same time and about the same situation. Making plans in a strategic situation requires thinking carefully before you act, taking into account what you think the people you are interacting with are also thinking about and planning. Because this kind of thinking is complex we need some sharp analytical tools in order to explain behaviour and predict outcomes in strategic situations – this is what game theory is for.

### 1.2 Describing strategic games

In order to be able to apply game theory a first step is to define the boundaries of the strategic game under consideration. Games are defined in terms of their rules. The rules of a game incorporate information about the players’ identity and their knowledge of the game, their possible moves or actions and their pay-offs. The rules of a game describe in detail how one player’s behaviour impacts on other players’ pay-offs. A player can be an individual, a couple, a family, a firm, a pressure group, the government, an intelligent animal – in fact any kind of thinking entity that is generally assumed to act rationally and is involved in a strategic game with one or more other players.

Players’ pay-offs may be measured in terms of units of money or time, chocolate, beer or anything that might be relevant to the situation. However, it
is often useful to generalise by writing pay-offs in terms of units of satisfaction or utility. Utility is an abstract, subjective concept and its use is widespread in economics. My utility from, say, a bar of chocolate is likely to be different from yours and anyway the two will not be directly comparable, but if we both prefer chocolate to pizza we will both derive more utility from the former. When a strategic situation is modelled as a game and the pay-offs are measured in terms of units of utility (sometimes called utils) then these will need to be assigned to the pay-offs in a way that makes sense from the player's perspectives. What usually matters most is the ranking between different alternatives. Thus if a bar of chocolate makes you happier than a pizza the number of utility units assigned to the former should be higher. The actual number of units assigned will not always be important. Sometimes it is simpler not to assign numbers to pay-offs at all. Instead we can assign letters or symbols to pay-offs and then stipulate their rankings. For example, instead of assigning a pay-off of, say, ten to a bar of chocolate and three to a pizza, we could simply assign the letter A to the chocolate and the letter B to the pizza and specify that A is greater than B (i.e. A > B). This can be quite a useful simplification when we want to make general observations about the structure of a game. However, in some circumstances the actual value of the pay-offs is important and then we need to be a bit more precise (see Chapter 5).

Rational individuals are assumed to prefer more utility to less and therefore in a strategic game a pay-off that represents more utility will be preferred to one that represents less. Note that while this will always be true about levels of satisfaction or pleasure it will not always be the case when we are talking about quantities of material goods like chocolate – it is possible to eat too much chocolate. Players in a game are assumed to act rationally if they make plans or choose actions with the aim of securing their highest possible pay-off (i.e. they choose strategies to maximise pay-offs). This implies that they are self-interested and pursue aims. However, because of the interdependence that characterises strategic games, a player’s best plan of action for the game, their preferred strategy, will depend on what they think the other players are likely to do.

The theoretical outcome of a game is expressed in terms of the strategy combinations that are most likely to achieve the players’ goals given the information available to them. Game theorists focus on combinations of the players’ strategies that can be characterised as equilibrium strategies. If the players choose their equilibrium strategies they are doing the best they can given the other players’ choices. In these circumstances there is no incentive for any player to change their plan of action. The equilibrium of a game describes the strategies that rational players are predicted to choose when they interact. Predicting the strategies that the players in a game are likely to choose implies we are also predicting their pay-offs.

Games are often characterised by the way or order in which the players move. Games in which players move at the same time or their moves are hidden are called simultaneous-move or static games. Games in which the players move in some kind of predetermined order are call sequential-move or dynamic games. These two types of games are discussed in the following sections.
Pay-offs, equilibrium and rationality

- **Pay-off**: measures how well the player does in a possible outcome of a game. Pay-offs are measured in terms of either material rewards such as money or in terms of the utility that a player derives from a particular outcome of a game.

- **Utility**: a subjective measure of a player's satisfaction, pleasure or the value they derive from a particular outcome of a game.

- **Equilibrium strategy**: a ‘best’ strategy for a player in that it gives the player his or her highest pay-off given the strategy choices of all the players.

- **Equilibrium in a game**: a combination of players’ strategies that are a best response to each other.

- **Rational play**: players choose strategies with the aim of maximising their pay-offs.

1.3 Simultaneous-move games

In these kinds of games players make moves at the same time or, what amounts to the same thing, their moves are unseen by the other players. In either case, the players need to formulate their strategies on the basis of what they think the other players will do. We are going to look at three examples: hide-and-seek; a pub managers’ game; and a penalty-taking game. The first of these is a hidden-move game and the second and third are simultaneous-move games. Both types of games are analysed using the pay-off matrix or the strategic form of a game. In the first and third games the interests of the players are diametrically opposed; if one wins the other effectively loses. Games like this are games of pure conflict. Often the pay-offs of the players in games of pure conflict add to a constant sum. When they do the game is a constant-sum game. Both Hide-and-seek and the penalty-taking game are constant-sum games. If the constant sum is zero the game is a zero-sum game. Most games are not games of pure conflict. There is usually some scope for mutual gain through coordination or assurance. In such games there will be mutually beneficial or mutually harmful outcomes so that there are shared objectives. Games like this are sometimes called mixed-motive games. The pub managers’ game is a mixed-motive game.
1.3.1 Hide-and-seek

Hide-and-seek is played by two players called Robina and Tim. Robina chooses between only two available strategies: either hiding in the house or hiding in the garden. Tim chooses whether to look for her in the house or the garden. He only has 10 minutes to find Robina. If he looks where she is hiding (either the house or the garden) he finds her within the allotted time otherwise he does not. If Tim finds Robina in the time allotted he wins €50, otherwise Robina wins the €50.

Matrix 1.1 shows how the game looks from Robina’s perspective. The figures in the cells of the matrix are her pay-offs in euros. In the first cell of Matrix 1.1, on the top row of the first column, the zero shows that if Robina hides in the house and Tim looks in the house she loses. In the second cell, reading across the matrix, the 50 indicates that if she hides in the house and Tim looks in the garden she wins €50. On the bottom row of the matrix the 50 in the first column indicates that if Robina hides in the garden and Tim looks in the house she wins the €50 but the zero in the second column shows that if she hides in the garden and Tim looks in the garden she loses.

Matrix 1.1 Robina’s pay-offs in hide-and-seek

<table>
<thead>
<tr>
<th></th>
<th>Tim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robina</td>
<td>look in house</td>
</tr>
<tr>
<td>hide in house</td>
<td>0</td>
</tr>
<tr>
<td>hide in garden</td>
<td>50</td>
</tr>
</tbody>
</table>

Matrix 1.2 shows how the game looks from Tim’s perspective. In Matrix 1.2 the pay-offs in the cells show that if Robina hides in the house and Tim looks in the house he finds her and wins the €50, but if he looks in the garden when she hides in the house he loses. Similarly, if Robina hides in the garden and Tim looks in the house he loses but if he looks in the garden when she hides in the garden he finds her and wins the €50.

Matrix 1.2 Tim’s pay-offs in hide-and-seek

<table>
<thead>
<tr>
<th></th>
<th>Tim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robina</td>
<td>look in house</td>
</tr>
<tr>
<td>hide in house</td>
<td>50</td>
</tr>
<tr>
<td>hide in garden</td>
<td>0</td>
</tr>
</tbody>
</table>
To analyse the game we need to show both players’ pay-offs in the same matrix. This is done in Matrix 1.3 which is the strategic form or pay-off matrix of hide-and-seek. It shows all the possible pay-offs of the players that result from all their possible strategy combinations. It is a convention that in each cell the pay-off of the player whose actions are designated by the rows of the matrix are written first. The pay-offs of the player whose actions are denoted in the columns are written second. So in this pay-off matrix Robina’s pay-offs are written first and her pay-offs and strategies are highlighted in blue. For example, the pay-offs in the cell in the top row of the first column are 0 to Robina and 50 to Tim. This shows that if Robina hides in the house and Tim looks in the house, Tim wins the €50 and Robina’s pay-off is zero. The cell in the bottom row of the first column shows that if Robina hides in the garden and Tim looks in the house, Robina wins the €50 and Tim’s pay-off is zero.

Matrix 1.3  The pay-off matrix for hide-and-seek

<table>
<thead>
<tr>
<th></th>
<th>look in house</th>
<th>look in garden</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robina</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hide in house</td>
<td>0, 50</td>
<td>50, 0</td>
</tr>
<tr>
<td>hide in garden</td>
<td>50, 0</td>
<td>0, 50</td>
</tr>
</tbody>
</table>

1.3.2 Pub managers’ game

In the pub managers’ game the players are two managers of different village pubs, the King’s Head and the Queen’s Head. Both managers are simultaneously considering introducing a special offer to their customers by cutting the price of their premium beer. Each chooses between making the special offer or not. If one of them makes the offer but the other doesn’t the manager who makes the offer will capture some customers from the other and some extra passing trade. But if they both make the offer neither captures customers from the other although they both stand to gain from passing trade. Any increase in customers generates higher revenue for the pub. If neither pub makes the discounted offer the revenue of the Queen’s Head is €7 000 in a week and the revenue to the Kings Head is €8 000. The pay-off matrix for this game is shown in Matrix 1.4 below which shows the pay-offs as numbers representing revenue per week in thousands of euros.
Following the convention already noted in section 1.3.1, the pay-offs of the player whose actions are designated by the rows are written first. So in this game the pay-offs of the manager of the Queen’s Head are written first and his strategies and pay-offs are highlighted in blue. The matrix shows that if the Queen’s Head manager makes the special offer his pay-off is 10 (i.e. €10 000) if the King’s Head manager also makes the offer, and 18 if he doesn’t. Similarly if the King’s Head manager makes the offer his pay-off is 14 if the Queen’s Head manager also makes the offer, and 20 if he doesn’t.

Exercise 1.2
In the pub managers’ game what are the pay-offs of the managers if neither of them makes the offer? What is the pay-off of the Queen’s Head manager if he doesn’t make the offer but the manager of the King’s Head does? What is the pay-off of the King’s Head manager if he doesn’t make the offer but the manager of the Queen’s Head does?

Exercise 1.3
What do you think will be the outcome of the pub managers’ game. What do you think the managers will do?

Give some thought to Exercise 1.3. Although we haven’t actually looked at how to solve games yet, the pub managers’ game has an equilibrium that you can probably work out just by using a little common sense. In Chapter 2 you will see how to solve games like this in a systematic way. You will then be able to check whether your intuition was correct.

In hide-and-seek and the pub managers’ game the pay-offs represent monetary sums and it was convenient to do this. But this won’t always be possible as the next game shows.
1.3.3 Penalty taking

In the penalty-taking game the two players are the striker taking the penalty and the goalkeeper. Let’s assume that it is the last minute of the game and the score is one all. If the striker scores his team will win the game and if the goalkeeper saves the penalty his team will secure an honourable draw. If the striker scores he will be covered in glory and if the goalkeeper saves the penalty it will be he who is covered in glory. This time the pay-offs cannot really be measured in terms of money – being covered in glory is not really quantifiable in this way. Instead the pay-offs are best represented in terms of levels of subjective satisfaction or utility.

We can assume that if the striker misses, his satisfaction level is zero and if he scores, the goalkeeper’s satisfaction level is zero. This is clearly a simplification. You might prefer to assign a negative score in these circumstances or even different low scores. You can do this but bear in mind that these scores are subjective representations and therefore the players’ pay-offs are not directly comparable, even if we wanted to make this kind of comparison, which we don’t. If the striker scores, his satisfaction level will be sky-high and similarly, the goalkeeper will feel sky-high if he saves the penalty. How do we record these sky-high satisfaction levels? Well here, what really matters is the ranking of the players’ pay-offs so we could arbitrarily assign them a value of anything between 1 and some incredibly high figure like 100 billion. But smaller numbers are easier to handle so here I will use a pay-off of 10 to represent sky-high utility. You may prefer to add a few noughts and you should feel free to do that. You might also prefer to allocate different scores between the players for sky-high utility – perhaps you think the striker will feel happier if he scores than the goalkeeper will if he saves the penalty. But remember the scores are not directly comparable so this would really be an unnecessary complication.

In order to construct the pay-off matrix that corresponds to these pay-offs we need to make some additional assumptions. First of all we can assume that the striker always kicks the ball on target so he either scores or the goalkeeper makes a save. Second we can simplify the players’ strategies by assuming that the striker can only kick to his right, his left or straight ahead, these are his strategy choices. Similarly the goalkeeper can only move to the striker’s left, his right or he can stand his ground in the centre of the goal. If the goalkeeper’s action mirrors the striker’s he saves the penalty otherwise the striker scores. With these pay-offs and simplifying assumptions the pay-off matrix for this penalty-taking game looks like the one in Matrix 1.5 (I have highlighted the strategies and pay-offs of the striker).
Notice that in the cells of Matrix 1.5 the pay-offs always add to the constant sum 10 since if one player's pay-off is 10 the other's is zero. Therefore the interests of the players, like those of Robina and Tim in hide-and-seek, are diametrically opposed (in hide-and-seek the equivalent constant sum is 50). In both these games there is only one winner and the other player is a loser. Games like penalty-taking and hide-and-seek are called *constant-sum games*. If the constant sum in question is zero then the game is a *zero-sum game*. But any constant-sum game can be represented as a *zero-sum game* by subtracting half the constant sum from every pay-off. To see this subtract 5 from all the pay-offs in Matrix 1.5 or 25 from all the pay-offs in Matrix 1.3. All constant or zero-sum games are games of pure conflict and their outcomes are sometimes difficult to predict (you will see why in Chapter 2, Section 2.4.3). However, games of pure conflict won't always be constant-sum games although they can usually be represented in this way.8

In the penalty-taking game left, centre and right are the pure strategies of the striker and the goalkeeper. If the striker decides that he is going to kick the ball to the left this would imply that he had chosen one of his pure strategies. Alternatively he might prefer to randomise between his pure strategies by, for instance, mentally throwing a dice before he runs up to kick the ball (or actually throwing a dice before running onto the pitch). He could kick to the left if the dice showed a 1 or a 2, to the right if it showed a 3 or a 4 and to the centre of the goal otherwise. If he did this the probability of him choosing any one of his three pure strategies would be \( \frac{1}{3} \). We could write this as \( \left( \frac{1}{3} \text{ left; } \frac{1}{3} \text{ centre; } \frac{1}{3} \text{ right} \right) \). Strategies that mix up a player's pure strategies in this way are called *mixed strategies*. Mixed strategies like these can be useful in games of pure conflict.
conflict like penalty taking, where one player doesn’t want the other to be able to predict their move. Mixed strategies are explained in more detail in Chapter 6.

**Mixed strategy**
- A mix of pure strategies determined by a randomisation procedure.

In each of the games we have looked at so far we have used numbers to represent the players’ pay-offs. If the ranking of the pay-offs is all that matters (as opposed to their absolute values) it is sometimes more convenient to write the players’ pay-offs as letters. Using letters means that actual numbers do not have to be assigned to pay-offs and this can be useful if you want to generalise the results of one piece of analysis to other similar but not identical games. This will be something that we will want to do in many of the chapters of this book (see for example Chapter 3, Section 2). In the penalty game we could generalise the pay-offs in this way by substituting the letter W for the number 10 on the assumption that W is greater than zero (W > 0). Although the resulting game in Matrix 1.5.1 looks a bit different from the one in Matrix 1.5, in all important respects it is the same since W > 0 (as noted beneath the matrix). The striker still prefers outcomes in which his chosen strategy is not matched by the goalkeeper and the opposite is true for the goalkeeper.

**Matrix 1.5.1** Taking a penalty 1 with non-numerical pay-offs

<table>
<thead>
<tr>
<th>striker</th>
<th>goalkeeper</th>
</tr>
</thead>
<tbody>
<tr>
<td>left</td>
<td>0, W</td>
</tr>
<tr>
<td>centre</td>
<td>W, 0</td>
</tr>
<tr>
<td>right</td>
<td>W, 0</td>
</tr>
</tbody>
</table>

W > 0

1.4 **Sequential-move or dynamic games**

In sequential-move games players make moves in some sort of order. This means one player moves first and the other player or players see the first player’s move and can respond to it. Some illustrative examples are:
A firm considering entry into a monopolised industry where the incumbent may start a price war if it does enter.

(ii) Chess.

(iii) A series of offers and counter offers made by a potential buyer and seller of a house.

(iv) A large firm, Apex, considering whether to launch an expensive advertising campaign which may be matched by its main rival, Convex.\(^9\)

(v) The leader of one country planning to invade another country.

(vi) A film star who is deciding whether or not to sue a newspaper.

(vii) A landowner who puts up a sign threatening to sue trespassers.

In each of these examples one of the players moves first and another sees the first player’s move before deciding how to respond. This means that the order of moves is important and the analysis of this type of game has to take this into account. It is not always easy to do this using pay-off matrices and therefore sequential games are usually analysed using game trees or extensive forms like the one in Figure 1.1.

Figure 1.1 is drawn to represent the example in (iv). In this version of that game the two firms, Apex and Convex, choose between launching an advertising game or not. Apex moves first but the success of Apex’s campaign depends on what Convex does. A, C\(_1\) and C\(_2\) represent the decision points in the game. Apex’s choices are represented by the two branches that are drawn coming from the decision point or node labelled A. As Apex moves first this point is the first decision point in the game, the first point at which any player makes a move. At this point Apex chooses between launch or not launch. Whatever Apex decides Convex sees Apex’s move and can respond. If Apex launches its campaign the game moves to C\(_1\) where Convex decides whether to launch its

![Figure 1.1 The extensive form or game tree of Apex and Convex's advertising game](image-url)
campaign or not knowing full well that Apex has launched its campaign. At $C_1$ Convex can respond aggressively by launching its own campaign or respond passively by doing nothing. If Apex decides not to launch the campaign then the game moves to $C_2$ where Convex decides whether to launch its own expensive advertising campaign or not.

The pay-offs represent the firm’s profits in thousands of euros and they are written on the far right of the diagram at the endpoints or terminal nodes of the game tree, with Apex’s pay-offs written first. It is a convention that the pay-offs are written in the same order as the players’ moves, i.e. the pay-off of the player who moves first, in this case Apex, is written first. The pay-offs will always be written next to the terminal nodes of the appropriate branches of the game tree that mark the end of the game. In this game Apex’s pay-off depends not only on its own initial move but also on Convex’s response. Convex’s pay-off similarly depends on Apex’s initial move as well as its own move at either $C_1$ or $C_2$. If Convex responds aggressively to Apex’s move, whatever it is, by launching its own campaign Apex’s profits will be lower than if Convex had not launched its campaign. But if Apex does launch its campaign and Convex responds aggressively Convex’s profits are also lower as Convex’s action throws both firms into a damaging advertising war. However, if Apex doesn’t launch its campaign Convex benefits most by launching its campaign. This is shown by the players’ pay-offs at the ends of branches of the game tree. To see this look at the player’s pay-offs. When Apex decides on launching the campaign, if Convex responds by launching its own campaign, Apex’s pay-off is 2 and so is Convex’s. But if Convex doesn’t launch its own campaign both firms are better off – Apex’s pay-off is 6 and Convex’s is 3.

**Exercise 1.4**

What is Apex’s pay-off if it doesn’t launch the campaign but Convex does? What is Convex’s pay-off in these circumstances?

**Exercise 1.5**

What is Apex’s pay-off if it doesn’t launch the campaign and Convex doesn’t either? What is Convex’s pay-off in these circumstances?
The answer to Exercise 1.6 is not obvious but it is worth having a think about. In Chapter 4 we will use extensive forms like the one in Figure 1.1 to resolve sequential-move games like this game. You will be then be able to check whether your intuition was correct.

### 1.5 Repetition

Games that are only played once by the same players are called one-shot, single-stage or unrepeated games. Games that are played by the same players more than once are known as repeated, multi-stage or n-stage games where n is greater than one. The strategies of the players in repeated games need to set out the moves they plan to make at each repetition or stage of the game. These kinds of strategies are called meta-strategies.

The penalty game is a game that is likely to be played by the same players more than once; the same players in teams tend to take the penalties. Suppose the penalty game in Matrix 1.5 was played six times by the same two players. The striker’s meta-strategy for this repeated game could be to kick to the left in the first two repetitions then to the centre of the goal then twice to the right and then to the centre again. We would write this as (left, left, centre, right, right, centre). Alternatively he could choose a mixed strategy by randomising between left, right and centre every time he went to kick the ball. If his mixed strategy prescribed that he played each of his pure strategies with a probability of one-third then over the course of the repeated game we would expect to see him kicking to the left, right and centre a third of the time each. Repeated games are analysed in Chapter 8 and in some of the repeated games we are going to look at the players play mixed strategies.

### 1.6 Cooperative and non-cooperative games

Whether a game is cooperative or not is a technical point. Essentially a game is cooperative if the players are allowed to communicate and any agreements they make about how to play the game as defined by their strategy choices are enforceable. Most of the games we will look at in this book are non-cooperative.
even though in some of them players choose between cooperating with each other or not (for example in the prisoners’ dilemma games in Chapter 3). But being able to choose to cooperate does not make a game cooperative in the technical sense as such a choice is not necessarily binding. Being able to enforce agreements makes the analysis of cooperative games very different from that of non-cooperative games. Because agreements can be enforced the players have an incentive to agree on mutually beneficial outcomes. This leads cooperative game theory to focus on strategies that are implemented in the players’ joint or collective interests. This is not the case in non-cooperative game theory where it is assumed that player’s act only in their own self-interest. Some bargaining games are cooperative in this technical sense and these as well as non-cooperative bargaining games are analysed in Chapter 9.

1.7 **N-player games**

N is the number of players in the game. If a game has two players then it is a 2-player game. But if there are more than two players then the game is an N-player game where N is greater than 2. Most of the games we will look at in this book are 2-player games. The greater the number of players involved in a game the more complex it is likely to be.

1.8 **Information**

The equilibrium strategies of the players will depend on what kind of information players have about each other. In some games players will be very well informed about each other but this will not be true in all games. The information structure of a game can be characterised in a number of ways (see, for example, Montet and Serra, 2003: 4–6). The categories used in this book are perfect information, incomplete information and asymmetric information. If information is perfect then each player knows where they are in the game and who they are playing. If information is incomplete then a pseudo-player called ‘nature’ or ‘chance’ moves in a random way that is not clearly observed by all or some of the players. If not all the players observe the chance move then the information is also asymmetric. When information is asymmetric not all players have the same information. Instead some player has private information.

In all the games in Chapters 2–4 the players have perfect information. This is unlikely in real life and if game theory is to be really useful it needs to incorporate imperfect information. You will see how to do this in Chapters 5, 6 and 7.
In the games analysed in these chapters one or more of the players is less than perfectly informed.

When information is not perfect there is uncertainty in one or more of the players' minds about where they are in a game or who they are playing. For the players this implies an extra element of risk. In risky situations the outcome is uncertain and this uncertainty is characterised by a probability distribution. In strategic games risk is incorporated in terms of the initial or prior beliefs of the players. In some situations the players may also be able to update their beliefs as and when they receive information (see Chapter 7). Risk is not unique to strategic games. It is also a feature of many situations where an individual's choice of action is not strategically related to that of anyone else. In these cases risk is non-strategic. You will see how to model non-strategic risk in Chapter 5.

Whether the situation is strategic or not, where risk is involved decision makers need to incorporate the relevant probabilities into their decision making. They do this by forming expectations about likely outcomes and rational decision makers are assumed to choose in order to maximise their expected pay-off. This is an average of all the possible pay-offs corresponding to a given choice. It is calculated by multiplying (or weighting) each pay-off by the probability that it will occur. If the pay-offs are written as units of money or even chocolate then this calculation generates an expected value in terms of either money or chocolate. If the pay-offs are written in terms of utility values then the calculation generates an expected utility. These two alternatives are discussed further in Chapter 5 but for the moment it may be helpful to note that expected utility is potentially the more useful measure as it can incorporate people's different attitudes to risk.

In this chapter you have learned about some of the basic ideas and concepts that are central to game theoretic analysis. Games and game theory were defined in terms of strategic interdependence and some game theoretic terminology was explained. You have seen that games can be divided into two main groups according to whether they involve simultaneous or sequential moves. Simultaneous-move games are represented using pay-off matrices or strategic forms. Sequential-move or dynamic games are usually represented by extensive forms or game trees. Simultaneous-move and sequential games can be played only once or they can be repeated. In the next two chapters you will learn how to model and predict outcomes in single-stage simultaneous-move games. Sequential-move games are analysed in Chapter 4. In Chapters 5 and 6 single-stage games with incomplete information are analysed. Repeated games are the subject of Chapter 8. Strategic games can be either non-cooperative or cooperative. Most of the games you will see in this book are non-cooperative. Cooperative games are considered in Chapter 9.
1.1
There are no explicitly right or wrong answers in this exercise. By way of an example an answer for (e) might go as follows: in the civil war the players are the two opposing factions. At least one of the factions needs to compromise in order for an agreement to be reached. If only one party compromises (or compromises more than the other) they lose out in the agreement but if neither compromises there will be no agreement and the war will continue (to the disadvantage of both). Interestingly scientists at the Santa Fe Institute in New Mexico have devised a game that models a scenario a bit like this to calculate how the probability of each party’s decision to fight in a civil conflict or to compromise changes as the terms of the proposed agreement change (Dispatch report, *Guardian*, 18 November 2003).

1.2
If neither manager makes an offer the manager of the Queen’s Head gets 7 and the manager of the King’s Head gets 8. If the Queen’s Head manager doesn’t make the offer but the manager of the King’s Head does the manager of the Queen’s Head gets 4. If the King’s Head manager doesn’t make the offer but the manager of the Queen’s Head does the manager of the King’s Head gets 6.

1.3
Both managers are better off making the offer whatever the other manager does so there is no reason to expect them not to make the offer. The most likely outcome seems to be that both managers will make the offer. This is actually the dominant strategy equilibrium of the game as you will see in Chapter 2.

1.4
In these circumstances Apex’s pay-off will be 3 and Convex’s pay-off will be 6.

1.5
In these circumstances Apex’s pay-off will be 4 and Convex’s pay-off will be 4.

1.6
If Apex launches it gets either 6 if Convex doesn’t launch or 2 if Convex does. As Convex gets 3 by not launching if Apex also launches but 2 otherwise it should not launch if Apex also launches. Apex is assumed to know this. If Apex doesn’t launch it gets at most 4. So if Apex believes that if it launches Convex will not launch Apex should launch. Don’t worry if this chain of logic is not altogether clear as sequential games like this will be examined in detail in Chapter 4.
1 Think of one or two examples of real-life situations that could be represented as games and describe them using game theoretic terminology such as player, pay-offs and strategies.

2 In the examples you have thought of, do the players move simultaneously or sequentially and are their moves hidden or seen?

1 What is meant by strategic interdependence?

2 How can player’s pay-offs in games be represented?


2 Schelling (1960: 150) defines a strategic game in terms of dependence of one person’s choice of action on what he expects another to do and a strategic move as an action by one person that influences another person’s choice by affecting their expectations of how the first person will behave.

3 Except for some very expensive luxury items and some necessities, the relationship between consumer demand and price is assumed to be negative, i.e. if price rises demand falls and vice versa. Thus to encourage more sales in an industry market prices need to fall (assuming that no other important factors, for instance advertising or consumer tastes, change).

4 Binmore (1990) distinguishes three additional purposes of game theoretic models: description, investigation and prescription.

5 The thinking and rationality assumptions are not always applicable in evolutionary games (see Chapter 6).

6 If you want to know more about utility most introductory economics and all intermediate microeconomic textbooks have a chapter explaining how the concept is used to analyse various types of human behaviour (see, for example, Dawson, 2001, Chapter 4 in Himmelweit et al., 2001).

7 Or bimatrix as there is more than one pay-off in each cell.

8 Since in zero-sum games the pay-off of one player is just the negative of the other’s, pay-off matrices for zero-sum games often only show the pay-offs of one of the players.

9 An oligopoly market dominated by only two large firms is called a duopoly.